Ph 12b Recitation Notes

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1 The Overall Picture

Here, I give a very brief overview of the QM topics covered in this class (minus all the math). For more details, see (in decreasing order of brevity) the previous recitation notes, lecture notes, and/or textbook(s). Effectively all of non-relativistic quantum mechanics comes down to solving the Schrödinger equation. Namely,

$$i\hbar\frac{\partial \Psi(\vec{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{x},t) + V(\vec{x})\Psi(\vec{x},t) \tag{1}$$

The probability that a particle will be measured between position a and b is given by

$$P(a \leq x \leq b, t) = \int_a^b |\Psi(x,t)|^2 dx \tag{2}$$

assuming that the wavefunction is normalized. This equation is typically solved by separation of variables which divides the equation into a number of ordinary differential equations. There are many common 1D examples such as the infinite square well/hill, the finite square well/hill, simple harmonic oscillator, a free particle, a dirac delta potential, the triangular potential, etc., and other common 3D examples such as the hydrogen atom (spherical inverse square force) or the 3D infinite square box, simple harmonic oscillator, etc. So long as the potential does not explicitly depend on time, the equation can be separated into a time-independent equation and a time phase factor. The time-independent equation can be written in terms of linear combinations of the (often infinite number of) stationary states (solutions to the time-independent Schrödinger equation) multiplied by their time phase factors. Solutions are either bound states (when $E < V_\infty \equiv \min_\Omega \left[\lim_{r\to\infty} V(r,\Omega)\right]$, which yield discrete energies E_n) or scattering states (when $E > V_\infty$, which yield continuous spectrum). Spectral transitions are given by the change in the energy between energy states and are vital to many fields such as atomic, molecular, and optical physics (AMO), or astronomy.

Quantum mechanics is often more conveniently written in the language of linear algebra. The key components of this language are the particle states (with "kets" $|v\rangle$ represented as column vectors), the dual of the particle states (with "bras" $\langle v|$, represented as row vectors, encoding both the state and the inner product operation), and operators that act on bras and kets (with hat symbols $\hat{O}$, represented as matrices). With this, we can rewrite the time-independent Schrödinger equation as an eigenvalue problem using the Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V$, where $\vec{p} = -i\hbar\vec{\nabla}$. Namely,

$$\hat{H}\,|\psi(t)\rangle = E\,|\psi(t)\rangle \tag{3}$$

When operators are Hermitian $\hat{O}^\dagger = \hat{O}$, they are "observables" in that they correspond to actual physical measurements, since they always yield real eigenvalues. Thus, you can think of Hermitian operators as making a measurement on the particle, and the eigenvalues are the returned values (if they are configured to be in the corresponding eigenstate). One can find that this implies that operators that do not commute cannot be simultaneously measured (with infinite precision) and yield an uncertainty relation (similar to the position-momentum Heisenberg uncertainty principle or the energy-time uncertainty relation).

While the bound states/discrete eigenvalues in one dimension are characterized by one discrete parameter (e.g., the n in E_n), there are three in 3D (e.g., n, l, and m in spherical coordinates). The l and m parameters correspond to discretizations of the total angular momentum and the direction of the angular momentum, respectively. Many particles have intrinsic spin (s), which can sometimes be half-integers rather than integer values. Generally, we can have either bosons (which are symmetric under identical particle state interchange, and thus have integer spin $s = 0, 1, 2, ...$) or fermions (which are anti-symmetric under identical particle state interchange, and thus have half-integer spin $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...$).

The rest of non-relativistic quantum mechanics is largely just applications (e.g. technical applications such as perturbation theory, or practical applications like introductory AMO or condensed matter). However, this is the general framework. Were you to continue to study quantum mechanics, you would learn more such non-relativistic applications, and possibly eventually relativistic quantum mechanics in the form of the Dirac equation and then quantum field theories (QFTs) such as quantum electrodynamics (QED; quantum electromagnetism), electroweak theory (EWT; electromagnetism with weak nuclear force) and quantum chromodynamics (QCD; the strong nuclear force).

2 Final Review Problems

We will focus our problem solving today on questions that could conceivably appear on the final exam. These are taken from prior exams, they are listed on Canvas.

2.1 2025 Final Problem 2: Measurement in a Different Basis

2.2 2018 Final Problem 1: Short Answer Problems (except part b and e)

2.3 2025 Final Problem 3: Short Questions

2.4 2018 Final Problem 4: The Rotation Operator (time permitting)